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Extended performance evaluation criteria for enhanced heat transfer surfaces: heat transfer through ducts with constant wall temperature

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Abstract

Extended performance evaluation criteria equations for enhanced heat transfer surfaces based on the entropy production theorem have been developed to include the effect of fluid temperature variation along the length of a tubular heat exchanger with constant wall temperature as a boundary condition. The equations originate from various design constraints and generalize the performance evaluation criteria (PEC) for enhanced heat transfer techniques obtained by means of first-law analysis. The general evaluation criteria add new information to Bejan's entropy generation minimization (EGM) method assessing two objectives simultaneously. The application of this more comprehensive treatment of PEC compared to previous references is illustrated by the analysis of heat transfer and fluid friction characteristics of 10 spirally corrugated tubes assessing the benefit of these tubes as an augmentation technique. The results for different design constraints show that the optimum rib-height-to diameter ratio (e/D) for these tubes is about 0.04. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Performance evaluation criteria; Enhanced heat transfer; Entropy generation minimization; Single-phase turbulent flow in ducts; Constant wall temperature

1. Introduction

The performance of conventional heat exchangers, for single-phase flows in particular, can be substantially improved by many augmentation techniques resulting in the design of high-performance thermal design systems. Heat transfer enhancement devices are commonly employed to improve the performance of an existing heat exchanger or to reduce the size and cost of a proposed heat exchanger. On the basis of the first-law analysis several authors [1–5] have proposed performance evaluation criteria (PEC) which define the performance benefits of an exchanger having enhanced surfaces, relative to a standard exchanger with smooth surfaces subject to various design constraints.

On the other hand, it is well established that the minimization of the entropy generation in any process leads to the conservation of useful energy. A solid thermodynamic basis to evaluate the merit of augmentation techniques by second-law analysis has been proposed by Bejan [6,7] developing the entropy generation minimization (EGM) method also known as "thermodynamic optimization". The ultimate purpose is to evaluate the advantage of a given augmentation technique by comparing the rates of entropy generation in an augmented duct and in a reference smooth one. Bejan and co-workers [6–9] have applied this method to the design of two augmentation techniques: rough surfaces and swirl promoters analyzing heat transfer from ducts with constant heat flux. The method of

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Nomenclature

A	heat transfer surface area (m^2)
c_p	specific heat capacity (J kg^{-1} K^{-1})
D	tube diameter (m)
e	ridge height (m)
h	specific entalpy (J kg^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
L	tube length (m)
$\dot{m}$	mass flow rate in tube (kg s^{-1})
N_t	number of tubes
P	pumping power (W)
p	pitch of ridging (m)
Δp	pressure drop (Pa)
$\dot{Q}_t$	heat transfer rate from tube (W)
$\dot{S}_{gen}$	rate of entropy generation (W K^{-1})
s	specific entropy (J kg^{-1} K^{-1})
T	temperature (K)
ΔT	wall-to-fluid temperature difference (K)
u_m	mean flow velocity (m s^{-1})
U	overall heat transfer coefficient (W m^{-2} K^{-1})
v	specific volume of fluid (m^3 kg^{-1})
W	mass flow rate in heat exchanger (kg s^{-1})
x	axial distance along the tube (m)

Greek symbols

α	heat transfer coefficient (W m^{-2} K^{-1})
β	helix angle of rib (deg)
ϑ	temperature difference between wall and fluid, $T_w - T$
μ	dynamic viscosity (Pa s)
ρ	fluid density (kg m^{-3})

Dimensionless groups

A_*	dimensionless heat transfer surface, A_R/A_S
B	$4St_S L_S/D_S$
D_*	dimensionless tube diameter, D_R/D_S
L_*	dimensionless tube length, L_R/L_S
f	Fanning friction factor, $2\tau_w/(\rho u_m^2)$
Nu	Nusselt number, $\alpha_i D/k_f$
N_S	augmentation entropy generation number, Eq. (9)
N_*	ratio of number of tubes, $N_{t,R}/N_{t,S}$
Pr	Prandtl number, $\mu c_p/k_f$
P_*	dimensionless pumping power, P_R/P_S
Q_*	dimensionless heat transfer rate, $\dot{Q}_R/\dot{Q}_S$
Re	Reynolds number, $\rho u_m D/\mu$
St	Stanton number, $\alpha_i/(\rho u_m c_p)$
ΔT_i^*	dimensionless inlet temperature difference between hot and cold streams, $\Delta T_{i,R}/\Delta T_{i,S}$
$u_{m,*}$	dimensionless flow velocity, $u_{m,R}/u_{m,S}$
W_*	dimensionless mass flow rate, W_R/W_S
β_*	$\beta/90$
ε_*	ratio of heat exchanger effectiveness, $\varepsilon_R/\varepsilon_S$
τ	dimensionless temperature difference, $\Delta T/T$
ϕ_o	irreversibility distribution ratio

Subscripts

f	fluid
i	inside
i	value at $x = 0$
m	mean value
R	rough tube
S	smooth tube
o	outside
o	value at $x = L$
w	wall

Bejan [6,7] does not include the effect of variation in fluid temperature similar to that present in tubular heat exchangers. Extended PEC equations including the fluid temperature variation along the heat transfer passage for heat transfer from ducts with constant heat flux have been developed by Zimparov [10].

Nag and Mukherjee [11] and Perez-Blanco [12] modified Bejan's entropy generation criterion by including the fluid temperature variation along heat transfer passage for the case of heat transfer from duct with constant wall temperature. Prasad and Shen [13,14] have proposed an evaluation method based on exergy analysis. The thermodynamic optimum is obtained by minimizing the exergy destruction presented by exergy destruction number (N_E). In addition, another criterion such as heat transfer improvement number (N_H) is introduced. These numbers permit a comparison of the effect of improved heat transfer with increased irreversibility.

The purpose of this paper is to extend the PEC equations discussed previously [10] for the important case of heat transfer from a duct with constant wall temperature. The effect of fluid temperature variation along the length of a tubular heat exchanger is taken into consideration and new information to Bejan's EGM method is added assessing two objectives simultaneously.

2. Equations based on the entropy production theorem

Consider the energy balance of the control volume of length dx of the duct with constant wall temperature T_w, Fig. 1,

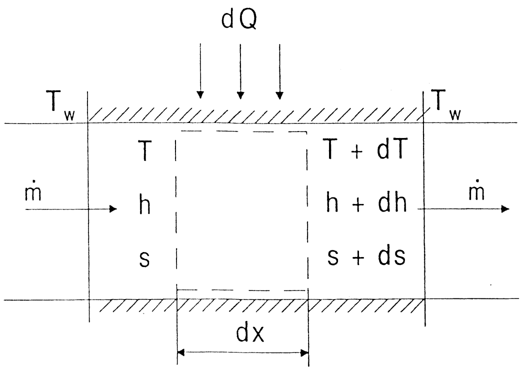

Fig. 1. Control volume for energy and entropy analyses.

$$\alpha_i\vartheta\pi D\,\mathrm{d}x = -\rho c_p u_m \frac{\pi D^2}{4}\mathrm{d}\vartheta. \tag{1}$$

Integrating Eq. (1) yields

$$\vartheta(x) = \vartheta_i \exp\left(-\frac{4St}{D}x\right), \tag{2}$$

where $\vartheta_i = T_w - T_i$ is the initial temperature difference. Considering an entropy balance in the same control volume, the rate of entropy generation is

$$\mathrm{d}\dot{S}_{gen} + \frac{\mathrm{d}\dot{Q}}{T_w} = \dot{m}\,\mathrm{d}s. \tag{3}$$

Assuming the fluid to be an ideal gas or to be incompressible, $\mathrm{d}h = c_p\,\mathrm{d}T$, and using the thermodynamic relation $T\,\mathrm{d}s = \mathrm{d}h - v\,\mathrm{d}p$ and $\mathrm{d}\dot{Q} = \dot{m}\,\mathrm{d}h$, Eq. (3) can be written as

$$\frac{\mathrm{d}\dot{S}_{gen}}{\mathrm{d}x} = \dot{m}c_p\frac{T_w - T}{TT_w}\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right). \tag{4}$$

Substituting the values $T_w = T(x) + \Delta T(x)$ and $\tau = \Delta T/T$

$$\frac{\mathrm{d}\dot{S}_{gen}}{\mathrm{d}x} = \dot{m}c_p\frac{\mathrm{d}T}{\mathrm{d}x}\frac{\Delta T}{T(T+\Delta T)} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right)$$
$$= \dot{m}c_p\frac{\mathrm{d}T}{\mathrm{d}x}\frac{\Delta T}{T^2(1+\tau)} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right). \tag{5}$$

For $\tau \ll 1$, Eq. (5) yields

$$\frac{\mathrm{d}\dot{S}_{gen}}{\mathrm{d}x} = \dot{m}c_p\frac{\Delta T}{T^2}\frac{\mathrm{d}T}{\mathrm{d}x} + \frac{\dot{m}}{\rho T}\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right), \tag{6}$$

where $T(x) = T_w - \vartheta_i\exp(-\frac{4St}{D}x)$, $\Delta T(x) = T_w - T(x) = \vartheta_i\exp(-\frac{4St}{D}x)$, and $\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{4St}{D}\vartheta_i\exp(-\frac{4St}{D}x)$. Integrating along the length of the duct

$$\dot{S}_{gen} = \dot{m}c_p\left[-\frac{\vartheta_i\exp\left(-\frac{4St}{D}L\right)}{T_w - \vartheta_i\exp\left(-\frac{4St}{D}L\right)} + \frac{\vartheta_i}{T_w - \vartheta_i} - \ln\frac{T_w - \vartheta_i\exp\left(-\frac{4St}{D}L\right)}{T_w - \vartheta_i}\right] + \frac{32\dot{m}^3}{\rho^2\pi^2}\frac{f}{D^5}\left[-\frac{D}{4St\,T_w}\left(\ln\frac{T_i}{T_o} - 4St\,L/D\right)\right],$$

or

$$\dot{S}_{gen} = \dot{Q}_t\frac{\vartheta_o}{T_iT_o} + \frac{32\dot{m}^3 f}{\rho^2\pi^2 D^5}\frac{L}{T_w}, \tag{7}$$

where $-\mathrm{d}p/\mathrm{d}x = 2f\rho u_m^2/D$; $u_m = 4\dot{m}/(\rho\pi D^2)$; $\dot{Q}_t = \dot{m}c_p(T_o - T_i)$;

$$\ln\frac{T_o}{T_i} = \ln\left(1 + \frac{T_o - T_i}{T_i}\right) \approx \frac{T_o - T_i}{T_i}$$

and

$$\frac{D}{4St\,L}\frac{T_o - T_i}{T_i} = \frac{T_w - T_m}{T_o - T_i}\frac{T_o - T_i}{T_i} = \frac{\vartheta_m}{T_i} \ll 1.$$

For tubular full-size heat exchanger, $\dot{Q} = N_t\dot{Q}_t$; $A = \pi DLN_t$; $A_f = \frac{\pi D^2}{4}N_t$; $W = N_t\dot{m}$ and Eq. (7) becomes

$$\dot{S}_{gen} = \frac{\dot{Q}}{N_t}\frac{\vartheta_o}{T_iT_o} + \frac{32W^3 fL}{N_t^3\rho^2\pi^2D^5T_w}. \tag{8}$$

The first and the second terms on the right-hand side of Eq. (8) represent the entropy generation due to heat transfer across finite temperature difference and due to friction, respectively. Following Bejan [6,7] the thermodynamic impact of the augmentation technique is defined by the augmentation entropy generation number

$$N_S = \dot{S}_{gen,\,R}/\dot{S}_{gen,\,S}. \tag{9}$$

Augmentation techniques with $N_S < 1$ are thermodynamically advantageous since in addition to enhancing heat transfer they reduce the degree of irreversibility of the apparatus. Substituting Eq. (8) into (9), N_S can be rewritten as

$$N_S = \frac{N_T + \phi_o N_P}{1 + \phi_o}, \tag{10}$$

where

$$N_T = \frac{\left(\dot{S}_{gen,\,\Delta T}\right)_R}{\left(\dot{S}_{gen,\,\Delta T}\right)_S} = \frac{Q_* \vartheta_{o,\,R}}{N_* \vartheta_{o,\,S}} \frac{T_{o,\,S}}{T_{o,\,R}}, \tag{11}$$

$$\frac{T_{o,\,S}}{T_{o,\,R}} = \left[\frac{T_{i,\,S}}{T_{o,\,S}} + \frac{Q_*}{W_*}\left(1 - \frac{T_{i,\,S}}{T_{o,\,S}}\right)\right]^{-1}, \tag{12}$$

$$\frac{\vartheta_{o,\,R}}{\vartheta_{o,\,S}} = \exp\left[B\left(1 - \frac{St_R}{St_S}\frac{L_*}{D_*}\right)\right], \tag{13}$$

$$B = \frac{4St_S}{D_S} L_S,$$

$$N_P = \frac{\left(\dot{S}_{gen,\,\Delta P}\right)_R}{\left(\dot{S}_{gen,\,\Delta P}\right)_S} = \frac{W_*^3 L_*}{N_*^3 D_*^5} \frac{T_{w,\,S}}{T_{w,\,R}} f_R/f_S = P_* \frac{T_{w,\,S}}{T_{w,\,R}}. \tag{14}$$

When the standard heat transfer passage is known, the numerical value of the irreversibility distribution ratio, $\phi_o = (\dot{S}_{gen},\ \Delta P/\dot{S}_{gen},\ \Delta T)_S$, describes the thermodynamic mode in which the passage is meant to operate

$$\phi_o = \frac{32 W_S^3 f_S L_S}{N_{t,\,S}^3 \rho^2 \pi^2 D_S^5 T_{w,\,S}} \frac{N_{t,\,S} T_{i,\,S} T_{o,\,S}}{Q_S (T_w - T_o)_S}. \tag{15}$$

Having in mind that $\dot{Q}_S = W_S c_p (T_o - T_i)$; $W_S = \dot{m}_S N_{t,\,S}$; $\dot{m}_S = \rho u_{m,\,S} \pi D_S^2/4$, the irreversibility distribution ratio ϕ_o can be simplified

$$\phi_o = \frac{32 \dot{m}_S^2 f_S L_S}{\rho^2 \pi^2 D_S^5 T_{w,\,S} c_p} \frac{T_{i,\,S} T_{o,\,S}}{\vartheta_{o,\,S} (T_o - T_i)_S}$$

$$= \frac{2 f_S L_S}{D_S} \left(\frac{u_m^2}{c_p T_w}\right)_S \frac{T_{i,\,S} T_{o,\,S}}{\vartheta_{o,\,S} (T_o - T_i)_S}. \tag{16}$$

When an enhanced tube is being considered for replacement of a smooth one, there are many possible effects on performance. The design constraints imposed on the exchanger flow rate and velocity cause key differences among the possible PEC relations [4,5]. The increased friction factor due to augmented surfaces may require a reduced velocity to satisfy a fixed pumping power (or pressure drop) constraint. If the exchanger flow rate is held constant, it may be necessary to increase the flow frontal area to satisfy the pumping power constraint. However, if the mass flow rate is reduced, it is possible to maintain a constant flow frontal area at reduced velocity. In many cases the heat exchanger flow rate is specified and a flow rate reduction is not permitted. Despite of the fact that a large number of possible PEC can be defined [15], the PEC as suggested by Webb and Bergles [4,5] characterize nearly all the PEC and they will be considered below. The equations are developed for tubes of different diameters and heat transfer and friction factors based on the presentation format of performance data for enhanced tubes [16]. The relative equations for single-phase flow inside enhanced tubes are:

$$A_* = N_* L_* D_*, \tag{17}$$

$$P_* = W_* \Delta p_* = f_R/f_S\ D_* L_* N_* u_{m,*}^3 = \frac{W_*^3 L_*}{N_*^2 D_*^5} f_R/f_S, \tag{18}$$

$$Q_* = W_* \varepsilon_* \Delta T_i^*, \tag{19}$$

$$W_* = u_{m,*} D_*^2 N_* = \frac{Re_R}{Re_S} D_* N_*, \tag{20}$$

$$\Delta p_* = f_R/f_S \frac{L_*}{D_*} u_{m,*}^2 = f_R/f_S \frac{L_*}{D_*^3} \frac{Re_R^2}{Re_S^2} \tag{21}$$

2.1. Fixed geometry criteria (FG)

These criteria may be thought of as a retrofit situation, in which there is a one-for-one replacement of smooth tubes with enhanced ones of the same basic geometry, e.g., tube envelope diameter, tube length, and number of tubes for in-tube flow. The FG-1 case seek increased heat duty or overall conductance UA for constant exchanger flow rate. The pumping power of the enhanced tube exchanger will increase due to the increased fluid friction characteristics of the augmented surface. For these cases the constraints $\Delta T_i^* = 1$, $W_* = 1$, $N_* = 1$ and $L_* = 1$ require $Re_S = D_* Re_R$ and $P_* > 1$.

One of the most common and amply documented heat transfer augmentation techniques is the surface promoters or "in-tube roughness". Wall roughness has a negligible impact on the flow cross section and hydraulic diameter D_h and in many applications it can be assumed $D_* = 1$. Nevertheless, in some cases thinner walled tubes might be used as replacement to diminish the pressure drop and pumping power increase [17]. If the tube-side velocity is reduced, the values of St_S and St_R are calculated at different Reynolds numbers, St_S for Re_S and St_R for Re_R. Fig. 2, pertaining to the important area of internal, single-phase forced convection flow, demonstrates the friction factor and heat transfer coefficients. If the friction factor and Stanton number characterizing the smooth surface in the turbulent flow are fitted by

$$f_S = 0.079 Re^{-0.25} \quad \text{and} \quad St_S = 0.023 Re^{-0.2} Pr^{-0.6}$$

the ratios St_R/St_S and f_R/f_S can be expressed, Fig. 2, by

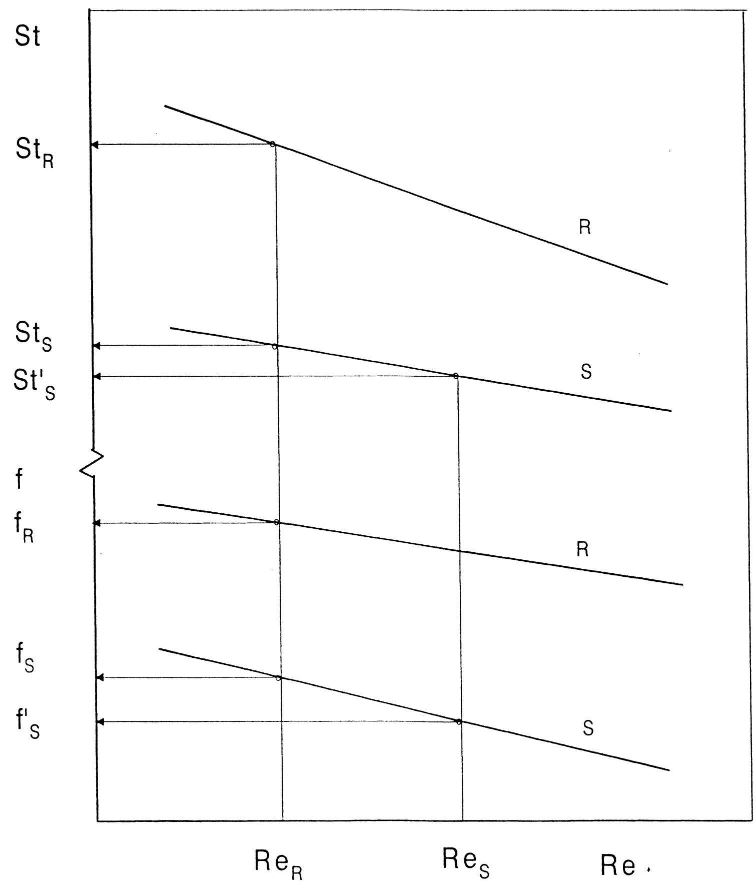

Fig. 2. The behavior of the Stanton number and friction factor vs. Reynolds number.

$$\frac{St_R(Re_R)}{St_S(Re_S)} = \frac{St_R(Re_R)}{St_S(Re_R)}\frac{St_S(Re_R)}{St_S(Re_S)} = \frac{St_R}{St_S}\left(\frac{Re_R}{Re_S}\right)^{-0.2} = f(Re_R),$$

and

$$\frac{f_R(Re_R)}{f_S(Re_S)} = \frac{f_R}{f_S}\left(\frac{Re_R}{Re_S}\right)^{-0.25} = f(Re_R).$$

When the objective is increased heat duty $Q_* > 1$, this corresponds to the case FG-1a [4,5], $Re_S = D_* Re_R$ and the augmentation entropy generation number N_S, Eq. (10), becomes

$$N_S = \frac{1}{1+\phi_o}\left\{Q_* \exp\left[B\left(1 - \frac{St_R}{St_S}D_*^{-0.8}\right)\right] \times \left[\frac{T_{i,\,s}}{T_{o,\,s}} + Q_*\left(1 - \frac{T_{i,\,s}}{T_{o,\,s}}\right)\right]^{-1} + \phi_o \frac{f_R/f_S}{D_*^{4.75}}\right\} = f(Re_R). \tag{22}$$

If the objective is $U_R A_R > U_S A_S$ for $Q_* = 1$, the driving LMTD can be reduced. This case corresponds to FG-1b [4,5]. The constraints $N_* = 1$, $L_* = 1$, $W_* = 1$ and $Q_* = 1$ require $Re_S = D_* Re_R$ and $P_* > 1$. The objective is $\Delta T_i^* < 1$. The augmentation entropy generation number N_S, Eq. (10), becomes

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}D_*^{-0.8}\right)\right] + \phi_o \frac{f_R/f_S}{D_*^{4.75}}\right\} = f(Re_R). \tag{23}$$

The FG-2 criteria have the same objectives as FG-1, but require that the augmented tube unit operates at the same pumping power as the reference smooth tube unit. The pumping power is maintained constant by reducing both the tube-side velocity and the exchanger flow rate. The constraints are: $\Delta T_i^* = 1$, $N_* = 1$, $L_* = 1$, and $P_* = 1$ requiring $W_* < 1$ and $Re_R < Re_S$. When the objective is $Q_* > 1$, case FG-2a [4,5], the augmentation entropy generation number N_S yields

$$N_S = \frac{1}{1+\phi_o}\left\{Q_* \exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{0.073}D_*^{-1.145}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + Q_*(f_R/f_S)^{0.364}D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o\right\} = f(Re_R). \tag{24}$$

Recall also that the constraint $P_* = 1$ requires

$$Re_S = Re_R(f_R/f_S)^{0.364}D_*^{-0.727} = f(Re_R). \tag{25}$$

When the objective is $\Delta T_i^* < 1$, with additional constraint $Q_* = 1$, case FG-2b [4,5], the augmentation entropy generation number N_S is

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{0.073}D_*^{-1.145}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + (f_R/f_S)^{0.364}D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o\right\} = f(Re_R). \tag{26}$$

The case FG-2c, where the objective is $\Delta T_i^* < 1$ with the constraints $N_* = 1$, $L_* = 1$, $Q_* = 1$ and $\Delta p_* = 1$ (pressure drop fixed), is an extension of cases FG-2. The consequences are $W_* < 1$, $P_* < 1$, $Re_R < Re_S$ and the case corresponds to the case B [18]. The constraint $\Delta p_* = 1$ imposes

$$Re_S = Re_R(f_R/f_S)^{0.571}D_*^{-1.714} = f(Re_R), \tag{27}$$

and Eq. (10) yields

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{0.114}D_*^{-1.343}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + (f_R/f_S)^{0.571}D_*^{-2.714}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o(f_R/f_S)^{-0.571}D_*^{2.714}\right\} = f(Re_R). \tag{28}$$

The third criterion, FG-3 [4,5], attempts to reduce the pumping power for constant heat duty. The constraints and the consequences are $N_* = 1$, $L_* = 1$, $\Delta T_i^* = 1$, $Q_* = 1$, $W_* < 1$ and $Re_R < Re_S$. The objective is $P_* < 1$. In this case

$$Re_S = Re_R\left(\frac{f_R/f_S}{P_*}\right)^{0.364}D_*^{-0.727} = f(Re_R), \tag{29}$$

and Eq. (10) becomes

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}\left(\frac{f_R/f_S}{P_*}\right)^{0.073}D_*^{-1.145}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + \left(\frac{f_R/f_S}{P_*}\right)^{0.364}D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o P_*\right\} = f(Re_R). \tag{30}$$

2.2. Fixed flow area criteria (FN)

These criteria maintain fixed flow frontal area. For a shell-and-tube heat exchanger having constant outside diameter tubes, this means that the number of tubes and shell diameter are held constant. These criteria assess reduced surface area or reduced pumping power for constant heat duty. The objective of FN-1 case [4,5] is reduced surface area by reduced tube length, $L_* < 1$, for constant pumping power, $P_* = 1$. The additional constraints are $N_* = 1$, $Q_* = 1$, $\Delta T_i^* = 1$ requiring $W_* < 1$ and $Re_R < Re_S$. In this case, the constraint $P_* = 1$ imposes

$$Re_S = Re_R\left(\frac{f_R}{f_S}L_*\right)^{0.364}D_*^{-0.727} = f(Re_R), \tag{31}$$

and the augmentation entropy generation number is

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{0.073}D_*^{-1.727}L_*^{1.073}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + \left(\frac{f_R}{f_S}L_*\right)^{0.364}D_*^{-1.727}\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o\right\} = f(Re_R). \tag{32}$$

The objective of FN-2 case [4] (FN-3 [5]), is to reduce pumping power, $P_* < 1$, with constant heat duty, $Q_* = 1$, and flow rate $W_* = 1$. Other constraints are $N_* = 1$, $\Delta T_i^* = 1$ and consequently $L_* < 1$ and $Re_S = D_* Re_R$. The equation for the augmentation entropy generation number is

$$N_S = \frac{1}{1+\phi_o}\left\{\exp\left[B\left(1 - \frac{St_R}{St_S}D_*^{3.95}\left(\frac{P_*}{f_R/f_S}\right)\right)\right] + \phi_o P_*\right\} = f(Re_R). \tag{33}$$

2.3. Variable geometry criteria (VG)

In many cases, a heat exchanger is designed for a

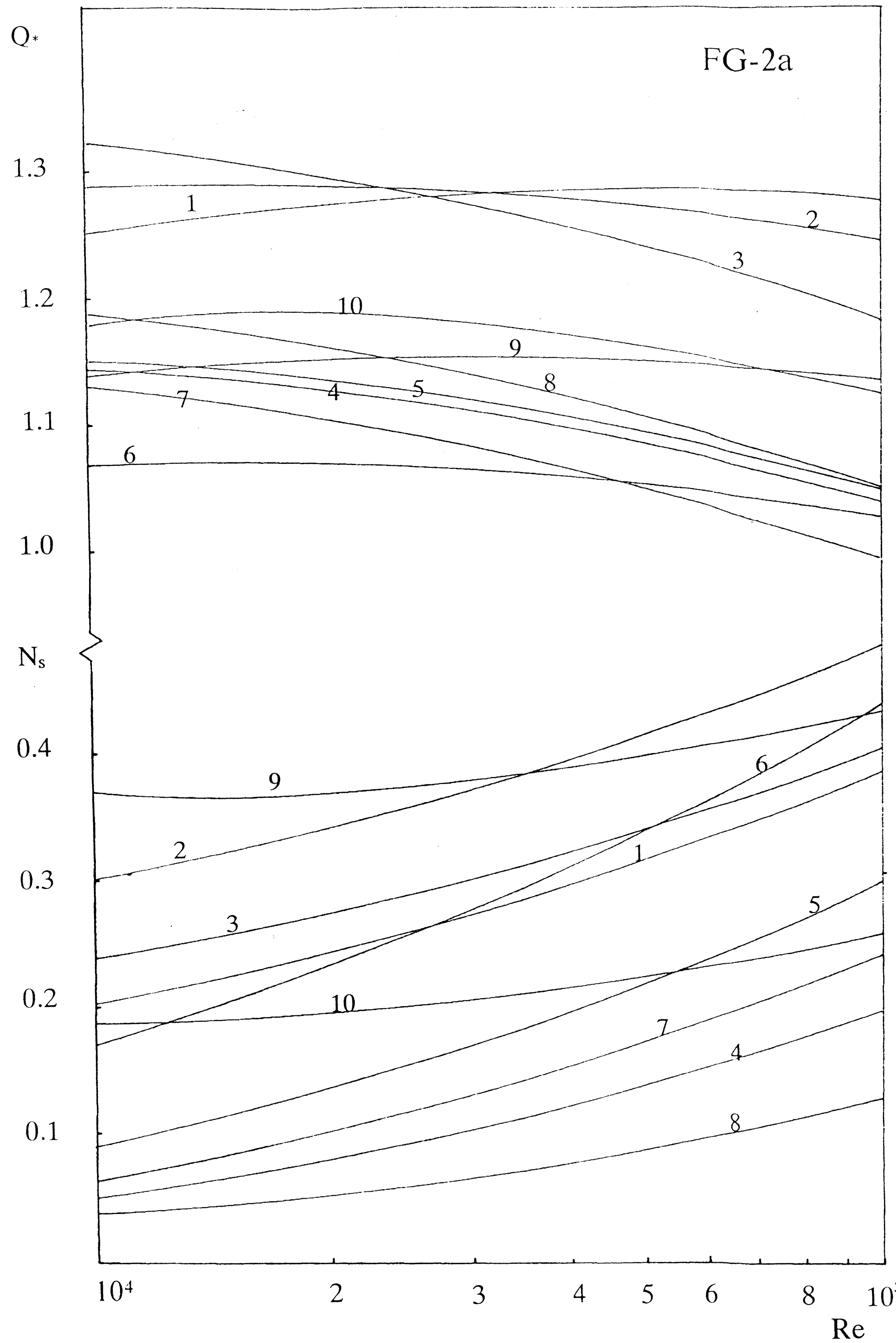

Fig. 3. Increased heat transfer rate and augmentation entropy generation number vs. Reynolds number.

required thermal duty with a specified flow rate. Because the tube-side velocity must be reduced to accommodate the higher friction characteristics of the augmented surface, it is necessary to increase the flow area to maintain constant flow rate. This is accomplished using a large number of tubes in parallel, or by using the same number of larger diameter tubes. All of the VG cases maintain $W_* = 1$ and permit the exchanger flow frontal area to vary in order to meet the pumping power constraint: $N_* > 1$, $L_* < 1$, $Re_R < Re_S$. Case VG-1 yields reduced surface area $A_* < 1$, for $Q_* = 1$, $P_* = 1$ and $\Delta T_i^* = 1$. The constraint $P_* = 1$ imposes

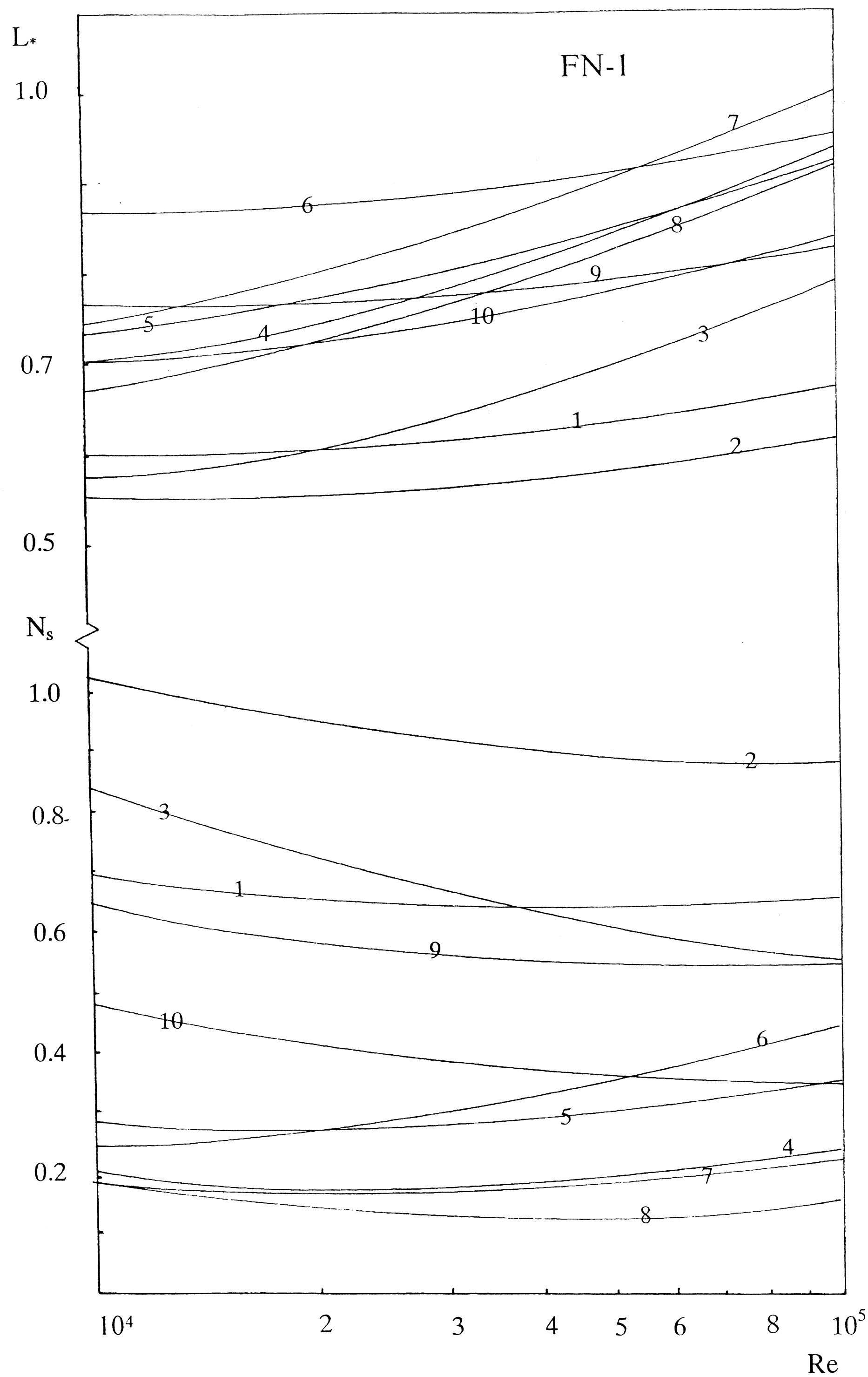

Fig. 4. Reduced tube length and augmentation entropy generation number vs. Reynolds number.

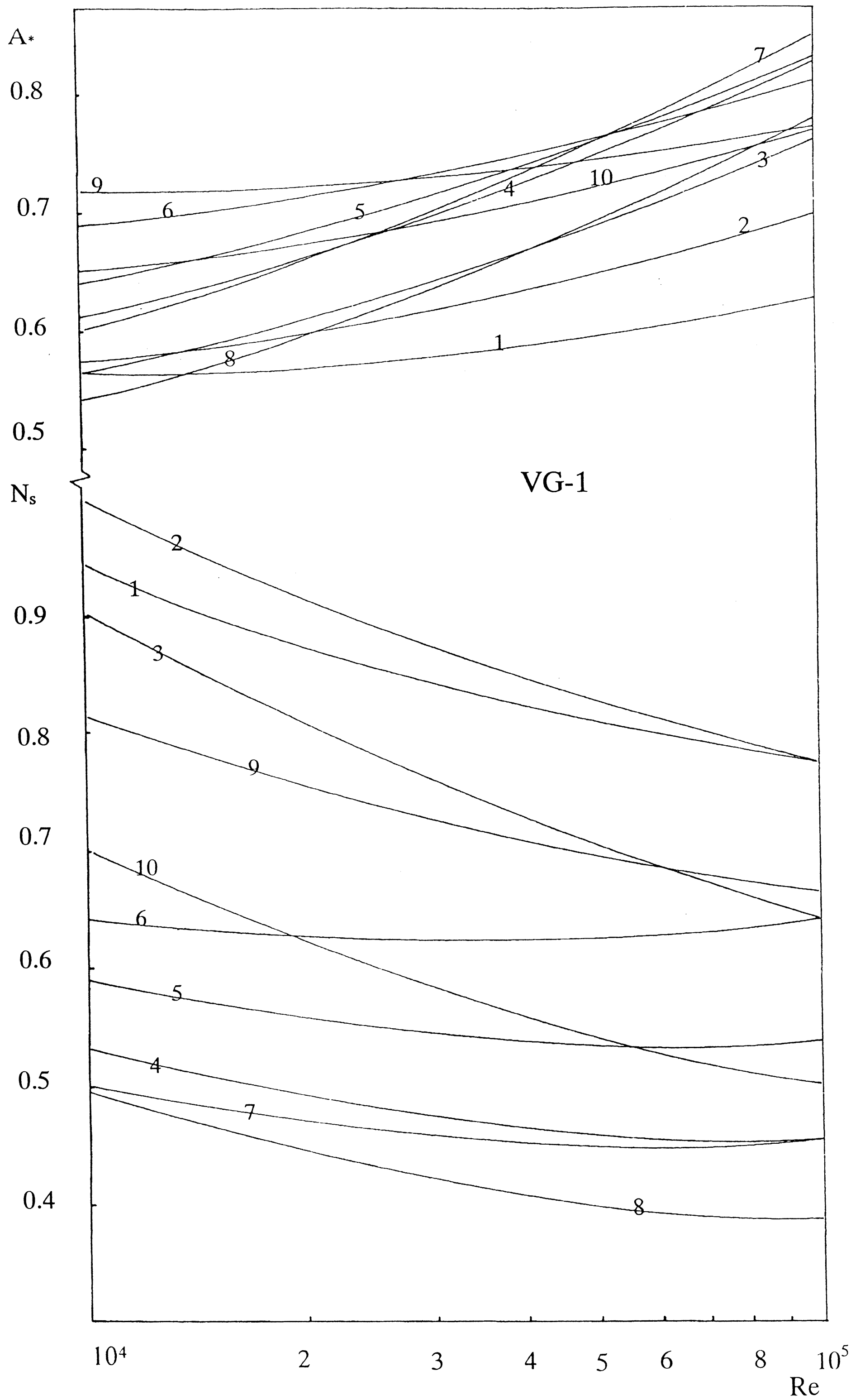

Fig. 5. Reduced surface area and augmentation entropy generation number vs. Reynolds number.

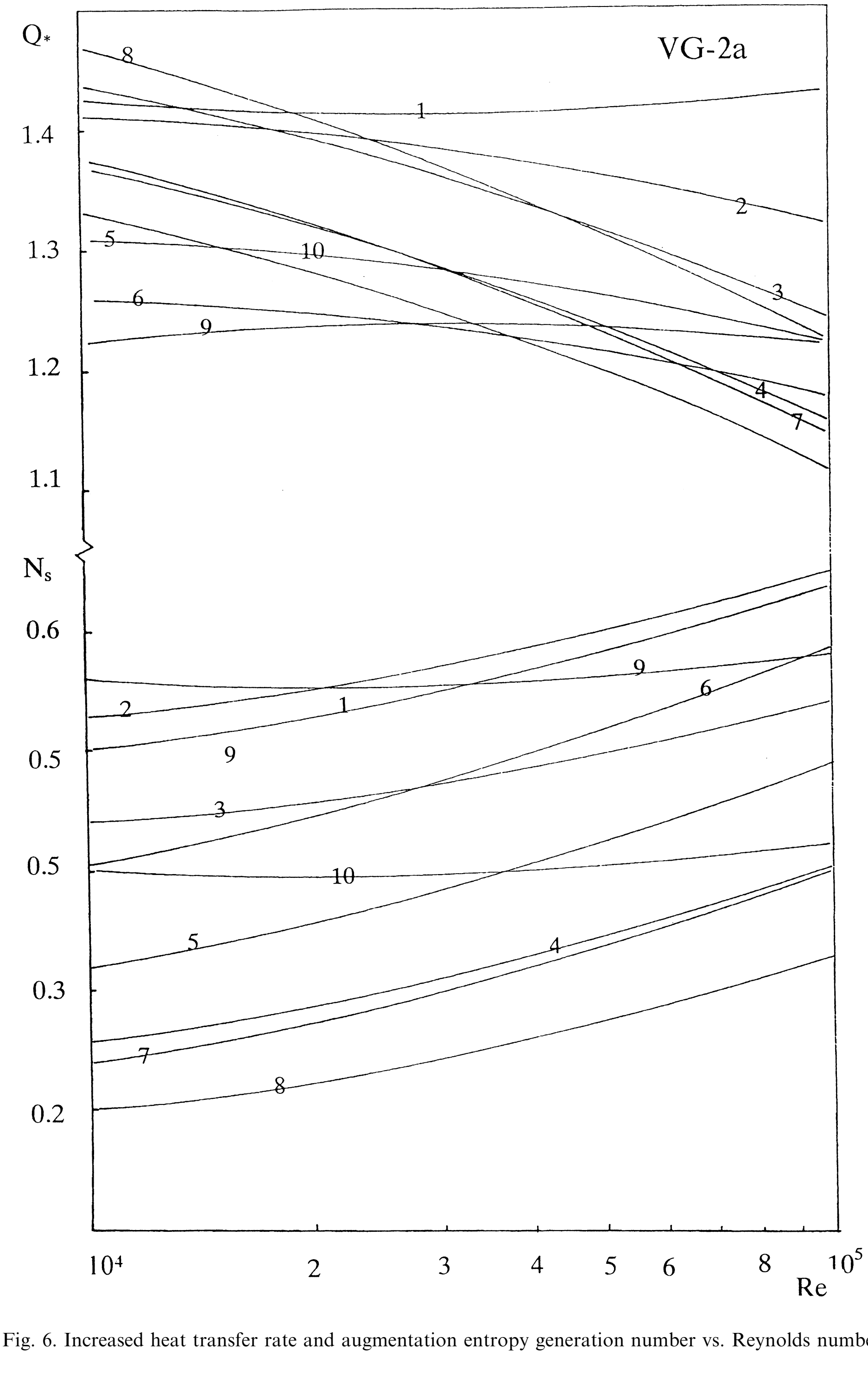

Fig. 6. Increased heat transfer rate and augmentation entropy generation number vs. Reynolds number.

$$Re_S = Re_R\left(\frac{f_R}{f_S}A_*\right)^{0.364} D_*^{-1.091} = f(Re_R), \tag{34}$$

and the augmentation entropy generation number becomes

$$N_S = \frac{1}{1+\phi_o}\left\{\left(\frac{f_R}{f_S}A_*\right)^{-0.364} D_*^{2.091} \exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{-0.291} A_*^{0.709} D_*^{-0.127}\right)\right] + \phi_o\right\} = f(Re_R). \tag{35}$$

Cases VG-2 [4,5] aim at increased thermal performance ($U_R A_R/U_S A_S$ or $Q_* > 1$) for $A_* = 1$ and $P_* = 1$. They are similar to the cases FG-2. When the objective is $Q_* > 1$, case VG-2a [4,5], an additional constraint is $\Delta T_i^* = 1$. The constraint $P_* = 1$ imposes

$$Re_S = Re_R (f_R/f_S)^{0.364} D_*^{-1.091} = f(Re_R), \tag{36}$$

and the augmentation entropy generation number N_S becomes

$$N_S = \frac{1}{1+\phi_o}\left\{Q_*(f_R/f_S)^{-0.364} D_*^{2.091} \cdot \exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{-0.291} D_*^{-0.127}\right)\right] \cdot \left[\frac{T_{i,S}}{T_{o,S}} + Q_*\left(1 - \frac{T_{i,S}}{T_{o,S}}\right)\right]^{-1} + \phi_o\right\} = f(Re_R). \tag{37}$$

When the objective is $\Delta T_i^* < 1$, with additional constraint $Q_* = 1$, case VG-2b [4,5], the augmentation entropy generation number N_S is

$$N_S = \frac{1}{1+\phi_o}\left\{(f_R/f_S)^{-0.364} D_*^{2.091} \cdot \exp\left[B\left(1 - \frac{St_R}{St_S}(f_R/f_S)^{-0.291} D_*^{-0.127}\right)\right] + \phi_o\right\} = f(Re_R). \tag{38}$$

Case VG-3 [4,5] aims to reduce the pumping power, $P_* < 1$ for $A_* = 1$, $Q_* = 1$ and $\Delta T_i^* = 1$. It is similar to case FG-3. The Reynolds numbers maintained in the comparable units are defined by

$$Re_S = Re_R\left(\frac{f_R/f_S}{P_*}\right)^{0.364} D_*^{-1.091} = f(Re_R), \tag{39}$$

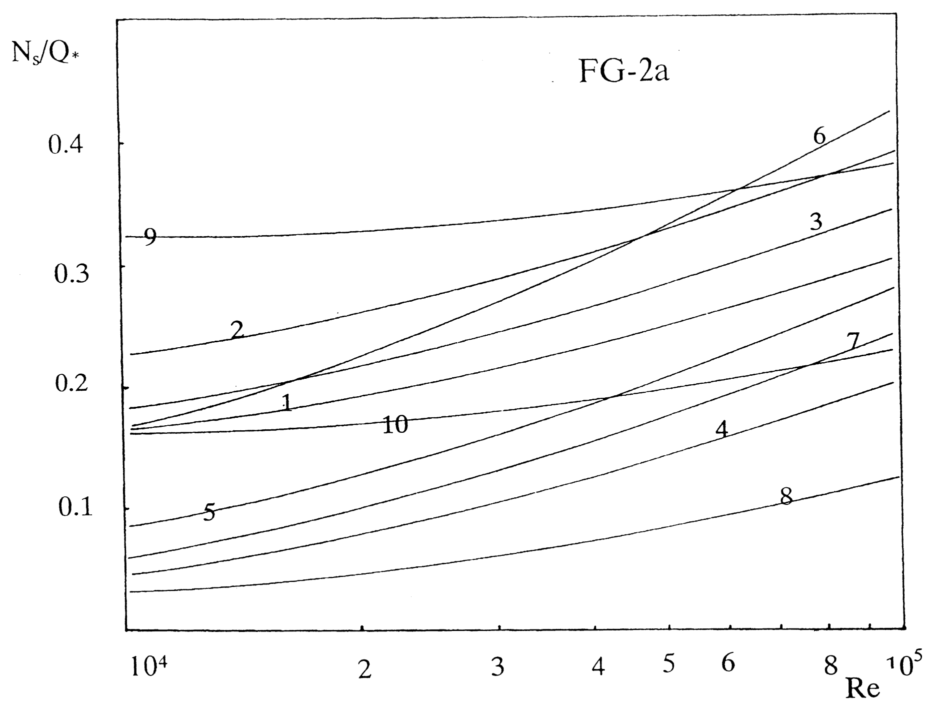

Fig. 7. The ratio N_S/Q_* vs. the Reynolds number.

and the equation for N_S has the form

$$N_S = \frac{1}{1+\phi_o}\left\{\left(\frac{f_R/f_S}{P_*}\right)^{-0.364} D_*^{2.091} \cdot \exp\left[B\left(1-\frac{St_R}{St_S}\left(\frac{f_R/f_S}{P_*}\right)^{-0.291} D_*^{-0.127}\right)\right] + \phi_o P_*\right\} = f(Re_R). \quad (40)$$

3. Application of PEC equation and discussion

The solution of the performance evaluation criteria equations described above requires algebraic relations which:

1. Define correlations for St (Nu) and f of the augmented surfaces as a function of Re.
2. Quantify performance objectives and design constraints. This means that the designer should define clearly his or her goal and then solve the equations corresponding to the algebraic relations [4] based on the first law of thermodynamics, to obtain the values of Q_*, A_* or P_* as a function of Re.
3. Calculate the irreversibility distribution ratio ϕ_o as a function of Re for the reference (smooth) passage.

The performance of heat exchangers, for single-phase flows in particular, can be improved by many augmentation techniques. The most popular and successful technique is augmentation through surface roughness. Transverse or helical repeated ribs are an especially attractive way of creating the surface roughness. The results of this study can be illustrated by the characteristics of spirally corrugated (roped) tubes for steam condensers obtained through several experimental programs [19–21]. The study of Ravigururajan and Bergles [22,23] based on the first-law analysis indicated that

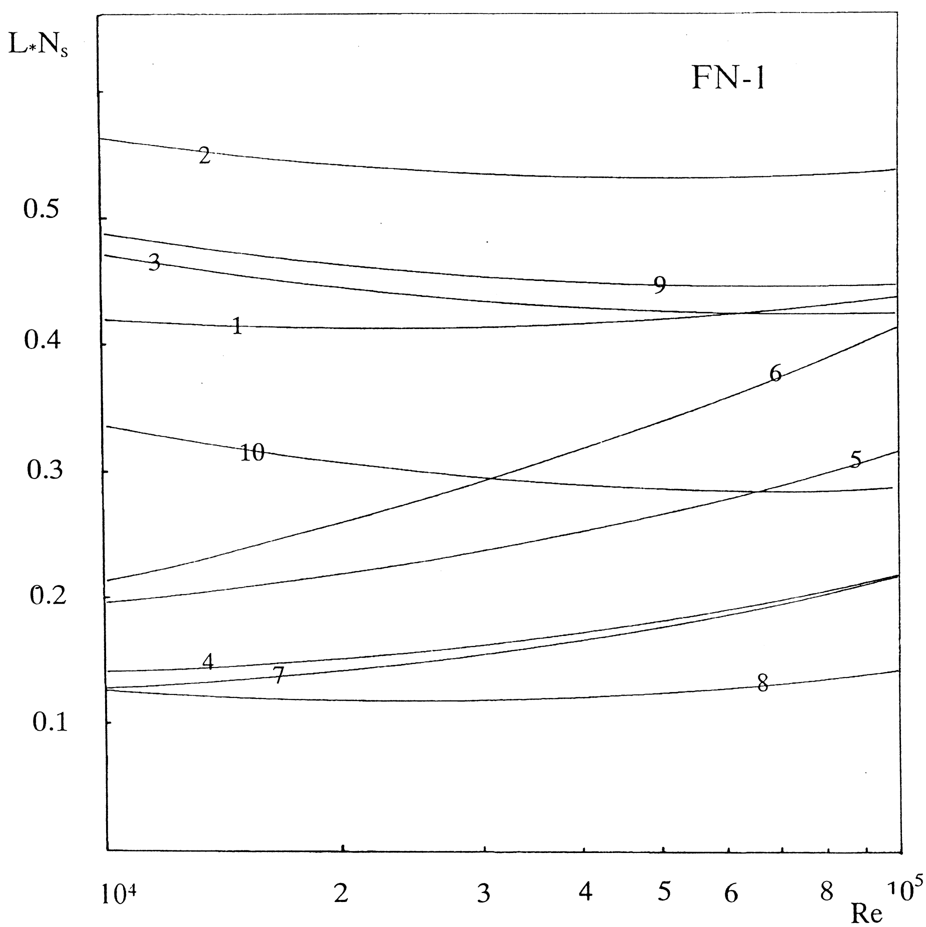

Fig. 8. The group $L_* N_S$ vs. the Reynolds number.

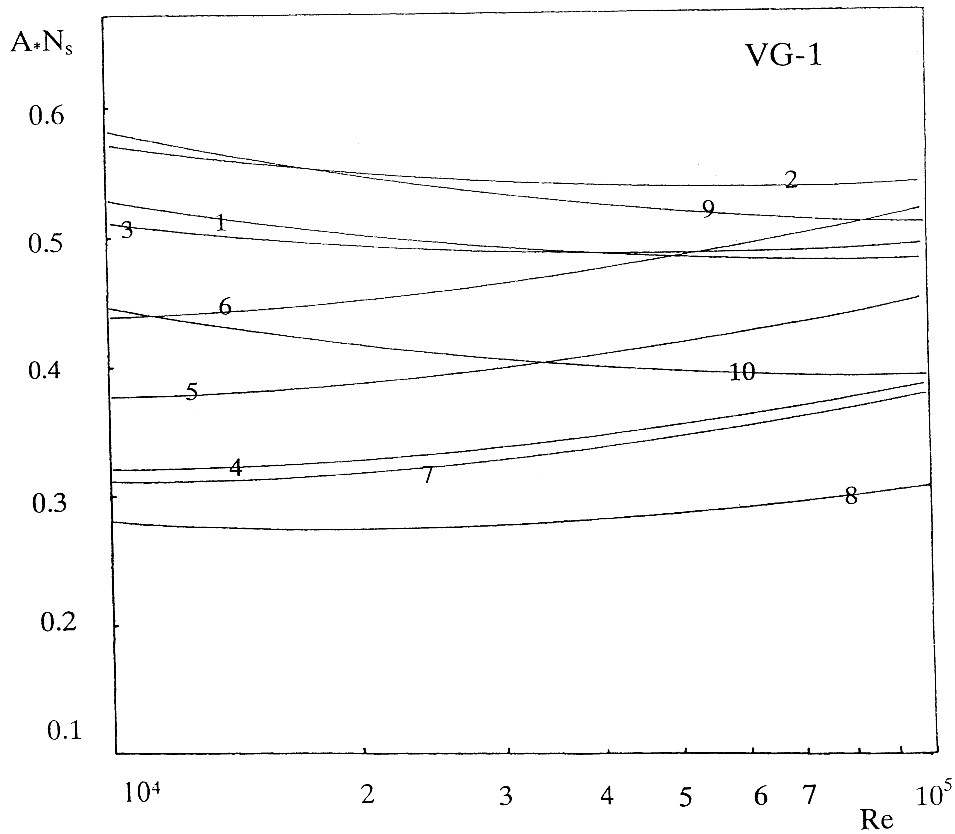

Fig. 9. The group $A_* N_S$ vs. the Reynolds number.

the optimum rib height-to-diameter ratio (e/D) for spirally indented tubes is around 0.02. This conclusion has been verified by using the PEC equations. The geometrical parameters of the tubes considered in this study are presented in Table 1.

The operating conditions of the reference (smooth) passage have been chosen as follows $T_{i,S} = 12°C$, $T_{o,S} = 21°C$, $Pr = 7.8$, $10^4 < Re < 10^5$. The corresponding values of ϕ_o are in the range $5.4 \times 10^{-4} < \phi_o < 1.7 \times 10^{-2}$ which shows that the channel is dominated by heat transfer irreversibility. The effect of the thermal resistance external to the surface is also taken into consideration by including external heat transfer coefficient $\alpha_{o,S} = 10{,}100$ W/m^2 K. The analysis includes the possibility that the augmented exchanger may have an enhanced outer tube surface, $E = \alpha_{o,R}/\alpha_{o,S}$. The values of E_o which have been assigned for each tube are presented in Table 1. The fouling resistances on both sides of the tube wall are neglected.

Fig. 3 represents the case FG-2a [4,5] where the objective is to increase the heat duty, $Q_* > 1$, of an existing heat exchanger, with the constraints for equal pumping power, $P_* = 1$, and heat transfer area, $A_* = 1$. The values of Q_*, as a function of Re_R are obtained following Webb's treatise on PEC [4]. First of all, it should be pointed out that every one of the corrugated tubes used in the unit leads to significant reduction of the rate of entropy generation, $N_S < 0.5$. As seen from the Fig. 3, the corrugated tubes 6, 14 [20] and 18 [21] have the smallest values of $N_S < 1$ but do not guarantee the largest heat transfer rate increase. On the other hand, the corrugated tubes 2a, 2b and 2c [19] guarantee largest values of Q_* but they do not guarantee minimum rate of entropy generation.

In the case FN-1 the goal is to reduce surface area for constant heat duty $Q_* = 1$ and pumping power $P_* = 1$, by reduced tube length, $L_* < 1$. The variations of L_* and N_S as a function of Re_R are shown in Fig. 4. The tubes which have the largest reduction of tube length $L_* < 1$ are 2a, 2b and 2c [19]. On the other hand, the tubes 6, 14 [20] and 18 [21] have the smallest value of $N_S < 1$ where the reduction of the rate of entropy generation is significant.

In the case VG-1 the objective is to reduce surface area $A_* < 1$ with $W_* = 1$ for $Q_* = P_* = 1$. Fig. 5 represents the variations of A_* and N_S with Re_R. In this case, the preferable tubes which have the largest reduction of heat transfer area $A_* < 1$ are 2a, 2b, 2c [19]

Table 1

Tube No.		Reference	D_o (mm)	D_i (mm)	e (mm)	p (mm)	e/D	p/e	β_*	E_o
1	2a		25.40	23.57	0.271	2.54	0.012	9.4	0.935	1.64
2	2b	[19]	25.40	23.57	0.393	6.35	0.017	16.1	0.840	1.29
3	2c		25.40	23.57	0.751	12.70	0.032	16.9	0.698	1.09
4	6		25.30	23.39	0.886	9.75	0.038	11.0	0.906	1.07
5	7	[20]	25.30	23.42	0.775	9.40	0.033	12.1	0.919	1.10
6	2200		24.61	21.90	0.439	6.35	0.020	14.5	0.941	1.25
7	14		25.38	22.00	0.947	13.46	0.043	14.2	0.878	1.04
8	18		27.27	25.20	1.022	10.95	0.041	10.7	0.919	1.07
9	33	[21]	27.56	25.62	0.447	6.55	0.017	14.7	0.952	1.20
10	34		27.62	25.78	0.628	8.48	0.024	13.5	0.938	1.08

and 18 [21]. On the other hand, the tubes 6, 7, 14 [20] and 18 [21] have the smallest value of $N_S < 1$.

Fig. 6 illustrates the case VG-2a where the objective is increased heat rate $Q_* > 1$ for $W_* = 1$ and $A_* = P_* = 1$. The best tubes having $Q_* > 1$ are 2a, 2b, 2c [19] and 18 [21], but the smallest values of N_S have the tubes 6, 7, 14 [20] and 18 [21] where the reduction of the rate of entropy generation is also significant.

The results shown in Figs. 4–6 imply that the evaluation and comparison of the heat transfer augmentation techniques should be made on the basis of both first- and second-law analysis. Thus, it is possible to determine the thermodynamic optimum in a heat exchanger by minimizing the augmentation entropy generation number compared with the relative increase of heat transfer rate $Q_* > 1$, or relative reduction of

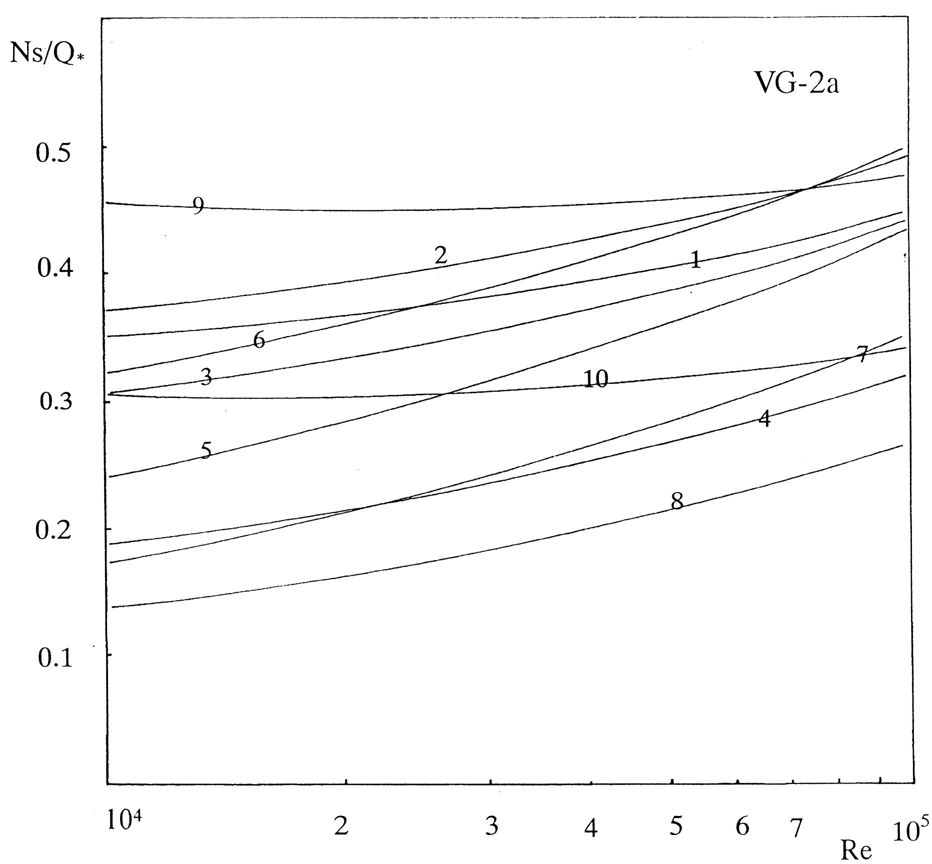

Fig. 10. The ratio N_S/Q_* vs. the Reynolds number.

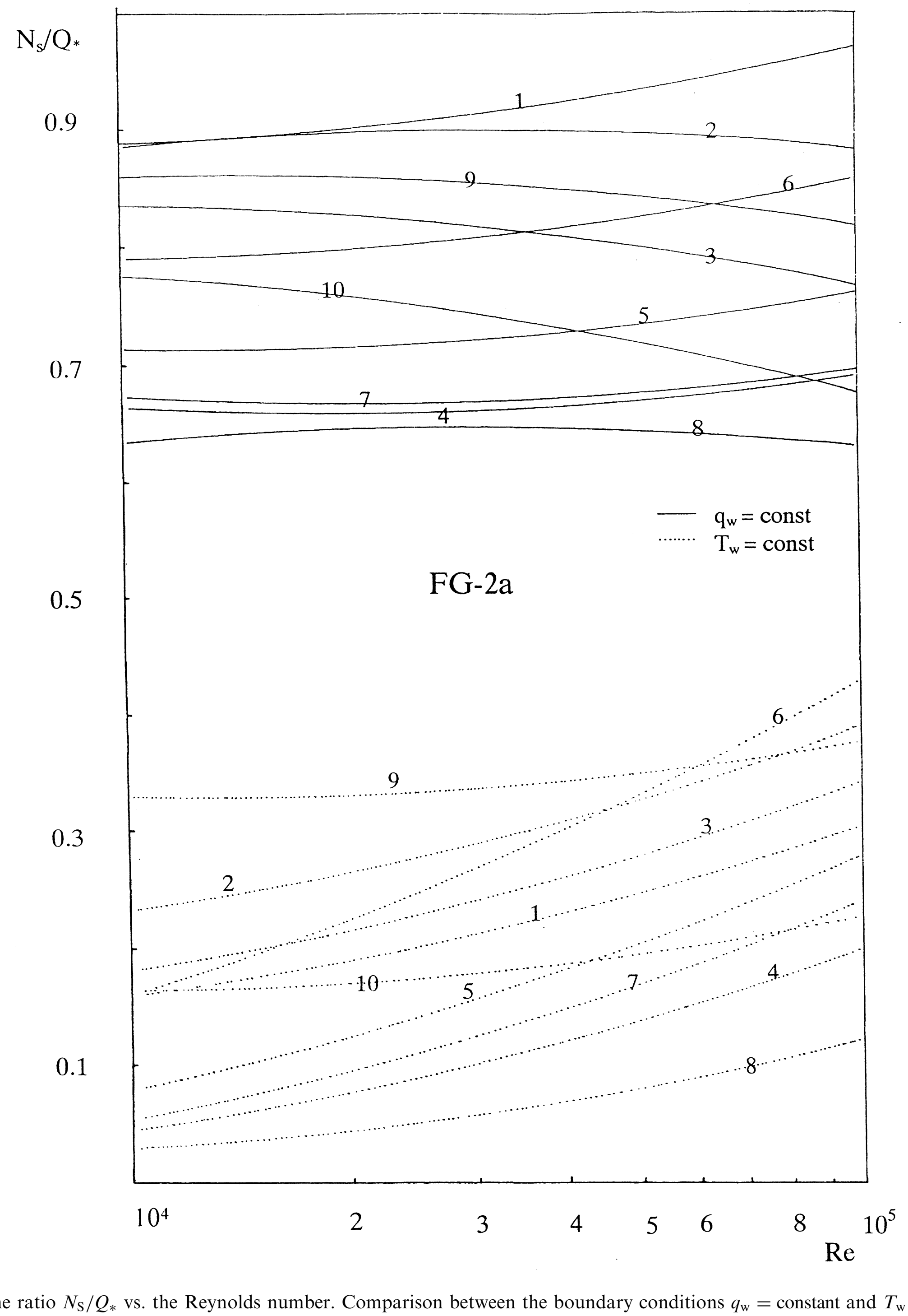

Fig. 11. The ratio N_S/Q_* vs. the Reynolds number. Comparison between the boundary conditions q_w = constant and T_w = constant.

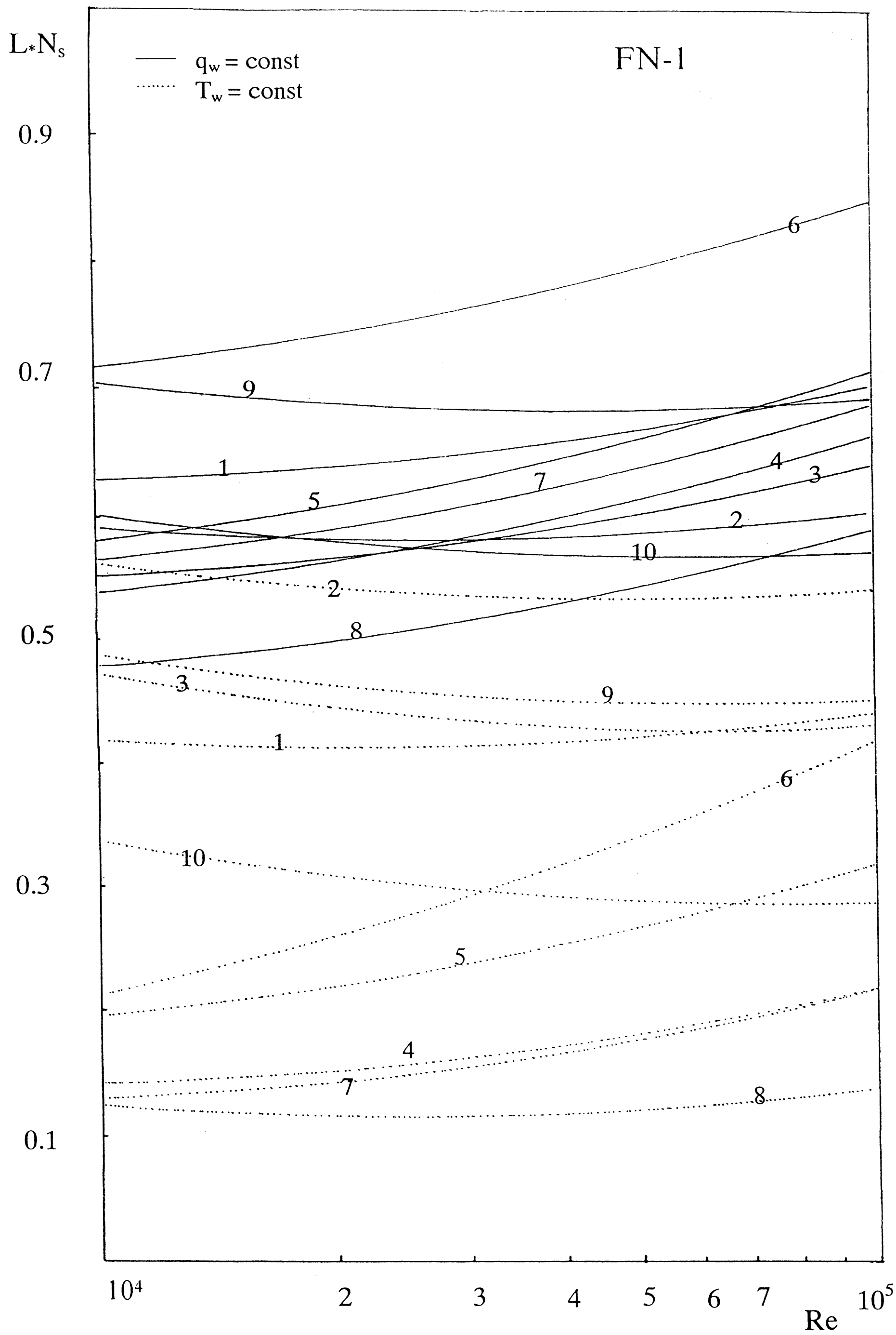

Fig. 12. The group $L_{*}N_{S}$ vs. the Reynolds number. Comparison between the boundary conditions q_w = constant and T_w = constant.

heat transfer area $A_* < 1$ ($L_* < 1$) or pumping power $P_* < 1$. Consequently, the ratios N_S/Q_*, N_SL_*, N_SA_*, $N_SP_* = f(Re)$ might be defined to connect the two objectives pursued by the first- and second-law analysis.

Figs. 7–10 show $N_S/Q_*–Re_R$ for the cases FG-2a and VG-2a, $N_SL_*–Re_R$ for the case FN-1 and $N_SA_*–Re_R$ for the case VG-1. For all the cases considered the best performance have one and the same tubes — 18 [21], 14, 6 [20] being far superior to others. The benefit of making use of tubes 18 [21] or 14, 6 [20] is significant. Therefore, the analysis using new performance evaluation criteria shows that the optimum rib-height-to-diameter ratio (e/D) for spirally corrugated tubes is about 0.04. Figs. 7–10 also show that for all the tubes considered an optimum Reynolds number, corresponding to minimum rate of entropy generation, can be obtained for lower *Re* number.

It is interesting to compare the results in this paper obtained for constant wall temperature boundary condition with the results [10] obtained for heat flux boundary condition. Figs. 11–14 show the performance evaluation ratios $N_S/Q_*–Re_R$ for the cases FG-2a and VG-2a, $N_SL_*–Re_R$ for the case FN-1 and $N_SA_*–Re_R$ for the case VG-1.

As seen for most of the cases considered, when the

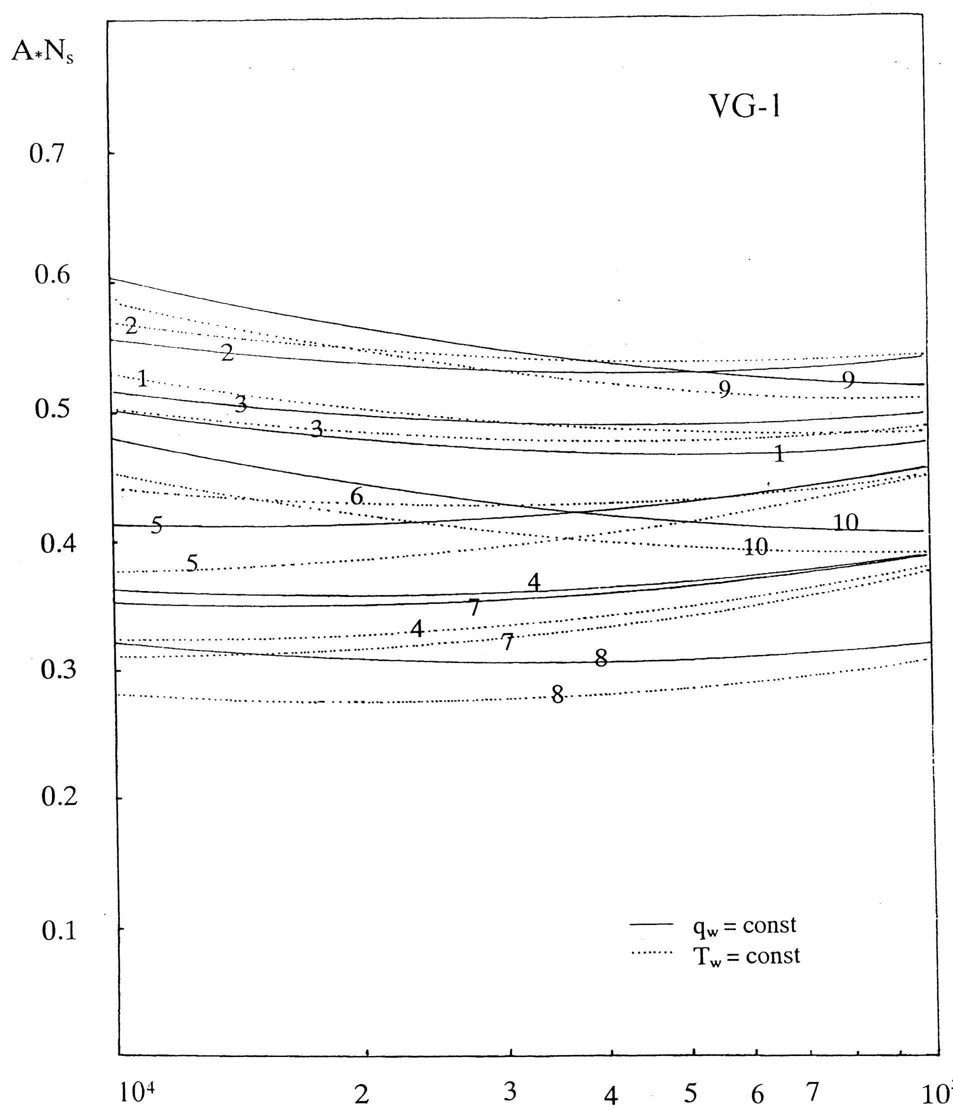

Fig. 13. The group A_*N_S vs. the Reynolds number. Comparison between the boundary conditions q_w = constant and T_w = constant.

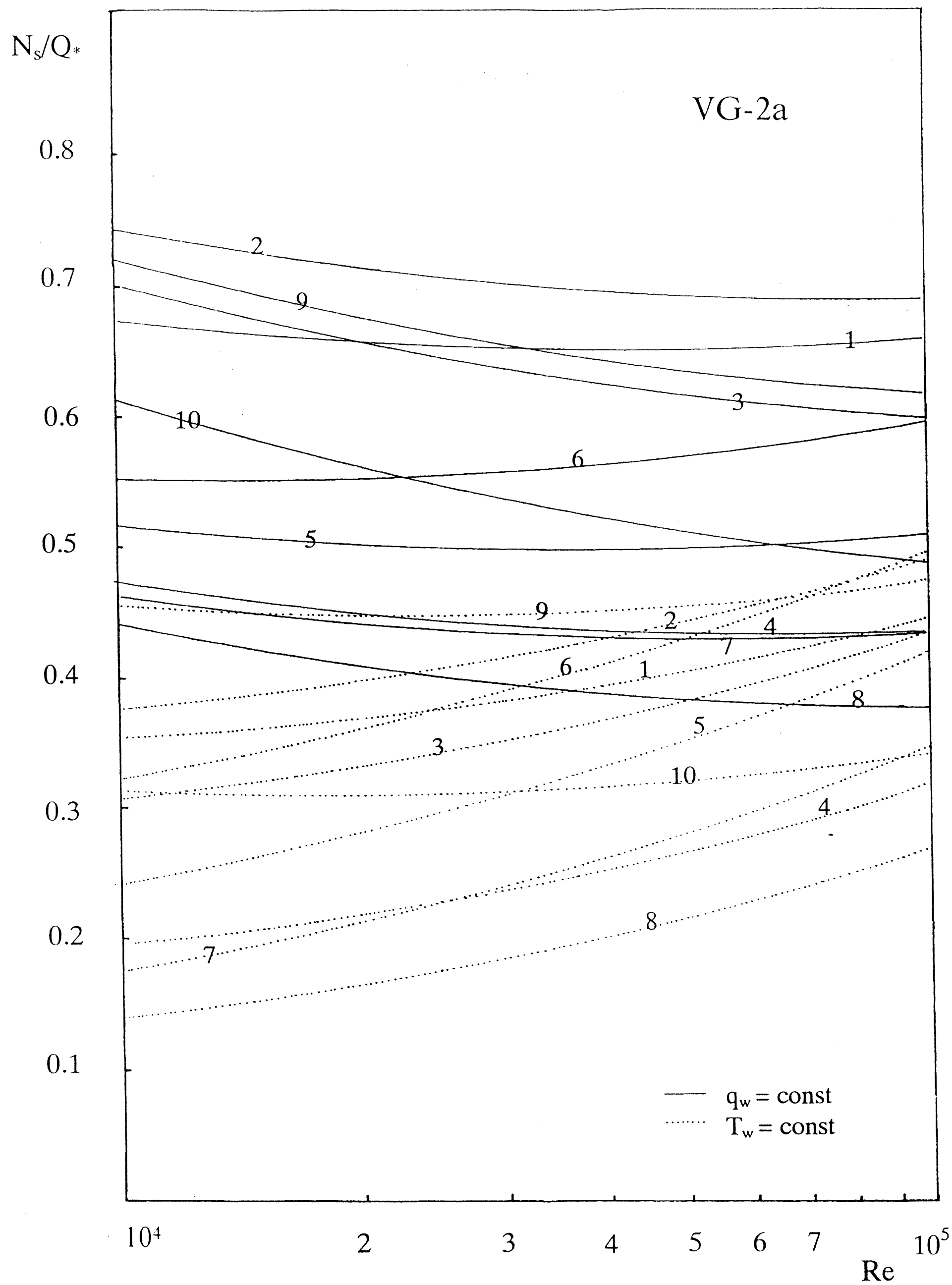

Fig. 14. The ratio N_S/Q_* vs. the Reynolds number. Comparison between the boundary conditions q_w = constant and T_w = constant.

heat exchanger operates at constant wall temperature and heat transfer augmentation techniques are utilized, this leads to greater merits compared to the benefits obtained when the heat exchanger operates at constant heat flux. Only for the case VG-1 is the difference between the two boundary conditions negligible.

4. Conclusions

The results of the present study can be summarized as follows:

1. Extended PEC equations have been developed to include the effect of fluid temperature variation

along the length of a tubular exchanger operating at constant wall temperature and to assess heat transfer enhancement techniques based on the entropy production theorem with various constraints imposed. These equations add new PEC for enhanced heat transfer surfaces developed by first-law analysis with criteria assessing the merits of augmentation techniques in connection with the entropy generation and exergy destruction.
2. The general evaluation criteria add new information to Bejan's EGM method assessing two objectives simultaneously. They may help to display inappropriate enhanced surfaces and assist the engineer to design better heat transfer equipment.
3. The heat transfer and fluid friction characteristics of 10 spirally corrugated tubes from three sources have been used to illustrate the application of the PEC equations. The results for different design constraints show that the optimum rib-height-to-diameter ratio (e/D) for spirally corrugated tubes is about 0.04.
4. A comparison between two boundary conditions shows that when an augmentation technique is utilized in a heat exchanger to improve its performance characteristics, more significant benefits can be obtained if it operates at constant wall temperature.

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